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Master function approach to solution of Dirac equation in (1+1)-spacetime

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Abstract : Scalar spinors are introduced in terms of master function and consequently, in terms of special functions, as solutions to Dirac equation It has been shown that the derived scalar spinors represent chiral supersymmetry algebra and unitary parasupersymmetry algebra of order p

Keywords : Dirac equation; parasupersymmetry; Shape invariance

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1. Introduction

In Ref. [1], a shape invariance relation on index n was obtained for associated differential equation of special functions, consequently we could introduce some quantum solvable systems in terms of master function $A(x)$. In Ref. [2], we also considered the shape invariance symmetry property of this differential equation with respect to parameter m, and then, introduced explicitly some other quantum solvable systems by special functions. For shape invariance with respect to parameter m it has been shown that special functions realize parasupersymmetry algebra of arbitrary order p. In this paper, it has been shown that shape invariance property is an approach to the solution of Dirac equation with a scalar field. In fact, using the ideas of supersymmetric quantum mechanics [3] and shape invariance [2,4] we obtain solution of Dirac equation for a scalar particle in (1+1)-spacetime. We show that these spinors are expressed in terms of special functions in mathematical physics, and represent $N = 1$ chiral supersymmetry algebra and parasupersymmetry algebra of order p.

We have recently introduced [1,2] the master function $A(x)$ as a polynomial of at most degree two, and non-negative weight function $W(x)$ in the interval $[a,b]$. The weight function $W(x)$ is determined in such a way that the expression $\frac{1}{W(x)}\frac{d}{dx}(A(x)W(x))$ becomes the polynomial of at most degree one. Also the interval $[a,b]$ is chosen by letting $A(x)W(x)$ and its derivatives vanish at the boundary points a and b. Then, it is easy to show that the polynomials $\Phi_n(x)$ of order n with the Rodrigues formula

$$\Phi_n(x)=\frac{a_n}{W(x)}\left(\frac{d}{dx}\right)^n\left(A^n(x)W(x)\right), \tag{1}$$

are orthogonal in the interval $[a,b]$, where constant a_n depends on the normalization procedure. Also these orthogonal polynomials satisfy the second order differential equation

$$A(x)\Phi_n''(x)=\frac{(A(x)W(x))'}{W(x)}\Phi_n'(x)$$
$$n\left(\frac{A(x)W'(x)}{W(x)}\right)'+\frac{n(n+1)}{2}A''(x)\ \Phi_n(x)=0. \tag{2}$$

By differentiating m times both sides of eq. (2), we obtain the following differential equation

$$A(x)\Phi_n^{(m)''}(x)+\frac{\left(A^{m+1}(x)W(x)\right)'}{A^m(x)W(x)}\Phi_n^{(m)'}(x)$$

$$(n-m \quad \frac{A(x)\left(A^m(x)W(x)\right)'}{A^m(x)W(x)}\Bigg)'$$

$$\frac{(n-m)(n-m+1)}{} A''(x)\Bigg]\Phi_n^{(m)}(x)=0, \quad (3)$$

with $\Phi_n^{(m)}(x)$ defined as

$$\Phi_n^{(m)}(x):=\left(\frac{d}{dx}\right)^m \Phi_n(x). \quad (4)$$

Differential eq. (3) is similar to differential eq. (2) with the same master function $A(x)$, but with a new weight function $A^m(x)W(x)$, and also $n-m$ instead of n. Then, for differential eq. (3), we have a polynomial solution of order $n-m$, that is $\Phi_n^{(m)}(x)$ has the following Rodrigues representation

$$\Phi_n^{(m)}(x)=\frac{a_n}{W(x)A^m(x)}\left(\frac{d}{dx}\right)^{n-m}\left(A^n(x)W(x)\right). \quad (5)$$

Finally, it is easily shown that change of function $\Phi_n^{(m)}(x)=(-1)^m A^{-m/2}(x)\Phi_{n,m}(x)$ for differential eq. (3) is culminated in the following associated differential equation, with master function $A(x)$ and weight function $W(x)$:

$$A(x)\Phi''_{n,m}(x)+\frac{\left(A(x)W(x)\right)'}{W(x)}\Phi'_{n,m}(x)$$

$$+\Bigg[-\frac{1}{2}(n^2+n-m^2)A''(x)+(m-n)\left(\frac{A(x)W'(x)}{W(x)}\right)'$$

$$\frac{m^2}{}\frac{\left(A'(x)\right)^2}{} \quad \frac{m}{}\frac{A'(x)W'(x)}{W(x)}\Bigg|\Phi_{n,m}(x)=0,$$

$$m=0,1,2,\ldots,n \quad (6)$$

with the solution

$$\Phi_{n,m}(x)=\frac{a_n(-1)^m}{W(x)A^{m/2}(x)}\left(\frac{d}{dx}\right)^{n-m}\left(A^n(x)W(x)\right). \quad (7)$$

Let us remind that in Ref. [2], we introduced the following operators for the given master function $A(x)$ and weight function $W(x)$

$$B_-(m)=\sqrt{A(x)}\frac{d}{dx}-\frac{\frac{m-1}{2}A'(x)}{\sqrt{A(x)}}$$

$$A_-(m)=-\sqrt{A(x)}\frac{d}{dx} \quad \frac{\frac{A(x)W'(x)}{W(x)}+\frac{m}{2}A'(x)}{\sqrt{A(x)}} \quad (8)$$

These raising and lowering operators $\underline{B}(m)$ and $\underline{A}(m)$ factorize the associated differential equation (6) in a shape invariant form with respect to index m as

$$B_-(m)A_-(m)\Phi_{n,m}(x)=E(n,m)\Phi_{n,m}(x),$$

$$A_-(m)B_-(m)\Phi_{n,m-1}(x)=E(n,m)\Phi_{n,m-1}(x), \quad (9)$$

where the spectrum $E(n,m)$ is

$$E(n,m)=-(n-m+1)$$

$$\left(\frac{A(x)W'(x)}{W(x)}\right)'+\frac{1}{2}(n+m)A''(x) \quad (10)$$

Using the following similarity transformation over the raising operator $B_-(m)$ and lowering operator $A_-(m)$ as

$$B(m)=A^{\frac{1}{4}}(x)W^{\frac{1}{2}}(x)B_-(m)A^{-\frac{1}{4}}(x)W^{-\frac{1}{2}}(x),$$

$$A(m)=A^{\frac{1}{4}}(x)W^{\frac{1}{2}}(x)A_-(m)A^{-\frac{1}{4}}(x)W^{-\frac{1}{2}}(x), \quad (11)$$

together with a change of variable x to θ by

$$\frac{dx}{d\theta}=\sqrt{A(x)}, \quad (12)$$

we obtain a new set of raising and lowering operators as follows

$$B(m)=\frac{a}{d\theta}+W_m(\theta),$$

$$A(m)=-\frac{d}{d\theta}+W_m(\theta), \quad (13)$$

which are adjoint of each other. Here, $W_m(\theta)$ is a superpotential and is written in terms of θ as

$$W_m(\theta)=-\frac{\frac{1}{2}\left(\frac{A(x)W'(x)}{W(x)}\right)+\frac{2m-1}{4}A'(x)}{\sqrt{A(x)}}\Bigg|_{x=x(\theta)} \quad (14)$$

in which prime denotes the dervative with respect to some real variable x. From equations of shape invariance (9) the following factorized equations for raising operator $B(m)$ and lowering operator $A(m)$ are obtained

$$B(m)A(m)\psi_{n,m}(\theta)=E(n,m)\psi_{n,m}(\theta)$$

$$A(m)B(m)\psi_{n,m-1}(\theta)=E(n,m)\psi_{n,m-1}(\theta). \quad (15)$$

Using Rodrigues formula (7) for associated special functions $\Phi_{n,m}(x)$, we obtain eigenstates $\psi_{n,m}(\theta)$ in terms of the master function as

$$\psi_{n,m}(\theta)=a_n(-1)^m\Big\{A^{(-2m+1)/4}(x)W^{-1/2}(x)$$

$$\left(\frac{d}{dx}\right)^{n-m}\left(A^n(x)W(x)\right)\Big\}_{x=x(\theta)}. \quad (16)$$

It is obvious that the first of eqs. (15) describes quantum systems with partner potential $V_{m,+}(\theta)$ $W_m^2(\theta)+\frac{dW_m(\theta)}{d\theta}$ and eigenstates $\psi_{n,m}(\theta)$, while that of the second describes quantum systems with partner potential $V_{m,-}(\theta)=W_m^2(\theta)-\frac{dW_m(\theta)}{d\theta}$ and eigenstates $\psi_{n,m-1}(\theta)$. Thus, eqs. (15) are precisely the Schrödinger like equations corresponding to the supersymmetric partner potentials $V_{m,\pm}(\theta)$. By appropriate selection of normalization coefficient a_n in (16), from the shape invariance (15) one can conclude that

$$B(m)\psi_{n,m-1}(\theta)=\sqrt{E(n,m)}\psi_{n,m}(\theta),$$

$$A(m)\psi_{n,m}(\theta)=\sqrt{E(n,m)}\psi_{n,m-1}(\theta). \tag{17}$$

Now, these preliminaries make us ready to refer to Dirac equation in presence of scalar field, and to introduce its solutions.

2. Dirac equation with a static scalar field in (1+1)-spacetime

Let us suppose that the static scalar field $W_m(\theta)$ has a finite energy solution for scalar field Lagrangian

$$\mathcal{L}_{W_m}=\frac{1}{2}\partial_\mu W_m\partial^\mu W_m-V(W_m),$$

in which $\mu=0(1)$ is ascribed to time (space) variable $t(\theta)$ respectively. Dirac Lagrangian with spacetime spinors $\Psi(\theta,t)$ is

$$L=i\overline{\Psi}(\theta,t)\gamma^\mu\partial_\mu\Psi(\theta,t)-\overline{\Psi}(\theta,t)W_m(\theta)\Psi(\theta,t) \tag{18}$$

in which,

$$\overline{\Psi}(\theta,t)=\Psi^\dagger(\theta,t)\gamma^0$$

and $W_m(\theta)$ is the static scalar field. Using the Pauli matrices, we choose $\gamma^0=\sigma^1$ and $\gamma^1=i\sigma^3$ as in Ref. [5]. For Lagrangian (18) the Dirac equation is obtained as

$$i\gamma^\mu\partial_\mu\Psi(\theta,t)-W_m(\theta)\Psi(\theta,t)=0. \tag{19}$$

If we choose time evolution of spinors $\Psi(\theta,t)$ as $\exp\left(-i\sqrt{E(n,m)}t\right)$, then by using eqs. (17), the spinors read

$$\Psi_{n,m}(\theta,t)=e^{-i\sqrt{E(n,m)}t}\Psi_{n,m}(\theta)$$

$$=e^{-i\sqrt{E(n,m)}t}\begin{pmatrix}\psi_{n,m-1}(\theta)\\ \psi_{n,m}(\theta)\end{pmatrix}\quad m=0,1,\ldots,n+1, \tag{20}$$

where we define $\psi_{n,-1}(\theta)=\psi_{n,n+1}(\theta)=0$. By multiplying both sides of eq. (19) from the left by γ^0, we get the time independent Dirac equation

$$H_D(m)\Psi_{n,m}(\theta)=\sqrt{E(n,m)}\Psi_{n,m}(\theta) \tag{21}$$

in which, Dirac operator $H_D(m)$ is

$$H_D(m)=\begin{pmatrix}0 & A(m)\\ B(m) & 0\end{pmatrix}. \tag{22}$$

It is clear that if the Dirac operator $H_D(m)$ is acted from the left on Dirac equation (21), we get the shape invariance eqs. (15) again. Therefore, as in the previous section, the shape invariant eqs. (15) yield exact energy eigenvalues and eigenfunctions for the Dirac equation involving a scalar field in 1+1 dimensions. These scalar fields $W_m(\theta)$ have scalar spinors that can be determined algebraically by exploiting shape invariance.

3. Dirac spinors as $N = 1$ chiral supersymmetry algebra representation

Defining chiral supersymmetry operators as [6]

$$Q\pm(m)=\frac{1}{2}(I\pm\gamma^5)H_D(m) \tag{23}$$

where $\gamma^5=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$, we get

$$H_D(m)=Q_+(m)+Q_-(m). \tag{24}$$

Since the operators $Q_+(m)$ and $Q_-(m)$ are nilpotent, *i.e.*, $Q_+^2(m)=Q_-^2(m)=0$, thus the square of the Dirac operator, together with $Q_+(m)$ and $Q_-(m)$ form the usual $N = 1$ supersymmetry algebra

$$H(m):=H_D^2(m)=\{Q_+(m),Q_-(m)\},$$

$$[Q_\pm(m),H(m)]=0. \tag{25}$$

Therefore, by virtue of the relations

$$Q_+(m)\Psi_{n,m}(\theta)=\sqrt{E(n,m)}\begin{pmatrix}\psi_{n,m-1}(\theta)\\ 0\end{pmatrix}$$

$$Q_-(m)\Psi_{n,m}(\theta)=\sqrt{E(n,m)}\begin{pmatrix}0\\ \psi_{n,m}(\theta)\end{pmatrix},$$

$$H(m)\Psi_{n,m}(\theta)=E(n,m)\Psi_{n,m}(\theta) \tag{26}$$

it becomes clear that spinors $\Psi_{n,m}(\theta)$ form a representation for $N = 1$ chiral supersymmetry algebra.

4. Scalar spinors as representation of shape invariance symmetry and unitary parasupersymmetry algebra

From the Refs. [2,7], we recall that the non-unitary parasupersymmetry algebra of order p, with parafermionic generators Q_1 and Q_2 and bosonic generator H, is defined as

$$Q_1^pQ_2+Q_1^{p-1}Q_2Q_1+\ldots+Q_1Q_2Q_1^{p-1}+Q_2Q_1^p=2pQ_1^{p-1}H, \tag{27a}$$

$$Q_2^pQ_1+Q_2^{p-1}Q_1Q_2+\ldots+Q_2Q_1Q_2^{p-1}+Q_1Q_2^p=2pQ_2^{p-1}H, \tag{27b}$$

$$Q_1^{p+1}=Q_2^{p+1}=0, \tag{27c}$$

$$[H,Q_1]=[H,Q_2]=0. \tag{27d}$$

If $Q_2=Q_1^\dagger$, then parasupersymmetry algebra (27) is a unitary parasupersymmetry algebra of order p. Now, we define the appropriate operators to express shape invariance on scalar spinors as

$$\tilde{B}_m = \begin{pmatrix} \sqrt{\dfrac{E(n,m)}{E(n,m-1)}} B(m-1) & 0 \\ 0 & B(m) \end{pmatrix},$$

$$\tilde{A}_m = \begin{pmatrix} \sqrt{\dfrac{E(n,m)}{E(n,m-1)}} A(m-1) & 0 \\ 0 & A(m) \end{pmatrix}. \tag{28}$$

By using eqs. (15), it is easy to show that shape invariance equations of spinors are

$$\tilde{B}_m \tilde{A}_m \Psi_{n,m}(\theta) = E(n,m) \Psi_{n,m}(\theta),$$

$$\tilde{A}_m \tilde{B}_m \Psi_{n,m-1}(\theta) = E(n,m) \Psi_{n,m-1}(\theta) \tag{29}$$

or

$$\tilde{B}_m \Psi_{n,m-1}(\theta) = \sqrt{E(n,m)} \Psi_{n,m}(\theta),$$

$$\tilde{A}_m \Psi_{n,m}(\theta) = \sqrt{E(n,m)} \Psi_{n,m-1}(\theta). \tag{30}$$

We indicate that spinors $\Psi_{n,m}(\theta), m = 0,1,\ldots,p$ can form the bases for a representation of para-supersymmetry [7] algebra of an arbitrary order p, with p as an integer, such that $1 \le p \le n+1$. Let us define parafermionic generators Q_1 and Q_2 and bosonic generator H as $2(p+1)\times 2(p+1)$ block matrices

$$(Q_1)_{mm'} = \tilde{A}_m \delta_{m+1,m'},$$

$$(Q_2)_{mm'} = \tilde{B}_{m'} \delta_{m,m'+1},$$

$$(H)_{mm'} = \tilde{H}_m \delta_{m,m'}, \qquad m,m' = 1,2,\ldots,p+1, \tag{31}$$

where $\tilde{H}_m$ too, similar to $\tilde{A}_m$ and $\tilde{B}_m$, are 2×2 matrices. With definition (31), automatically we have $Q_2 = Q_1^\dagger$ and $Q_1^{p+1} = Q_2^{p+1} = 0$. As in [2], Q_1, Q_2 and H form a unitary parasuperalgebra of order p, provided that $\tilde{A}_m$, $\tilde{B}_m$ and $\tilde{H}_m$ satisfy the relations

$$\tilde{B}_{p-1}\ldots\tilde{B}_2\tilde{B}_1\tilde{A}_1\tilde{B}_1 + \ldots + \tilde{B}_{p-1}\tilde{A}_{p-1}\tilde{B}_{p-1}\tilde{B}_{p-2}\ldots\tilde{B}_1$$
$$+ \tilde{A}_p\tilde{B}_p\tilde{B}_{p-1}\ldots\tilde{B}_1 = 2p\tilde{B}_{p-1}\tilde{B}_{p-2}\ldots\tilde{B}_1\tilde{H}_1,$$

$$\tilde{B}_p\ldots\tilde{B}_2\tilde{B}_1\tilde{A}_1 + \tilde{B}_p\ldots\tilde{B}_3\tilde{B}_2\tilde{A}_2\tilde{B}_2 + \ldots + \tilde{B}_p\tilde{A}_p\tilde{B}_p\tilde{B}_{p-1}\ldots\tilde{B}_2$$
$$= 2p\tilde{B}_p\tilde{B}_{p-1}\ldots\tilde{B}_2\tilde{H}_2,$$

$$\tilde{A}_1\ldots\tilde{A}_{p-1}\tilde{A}_p\tilde{B}_p + \tilde{A}_1\ldots\tilde{A}_{p-2}\tilde{A}_{p-1}\tilde{B}_{p-1}\tilde{A}_{p-1} + \ldots$$
$$+ \tilde{A}_1\tilde{B}_1\tilde{A}_1\tilde{A}_2\ldots\tilde{A}_{p-1} = 2p\tilde{A}_1\tilde{A}_2\ldots\tilde{A}_{p-1}\tilde{H}_p,$$

$$\tilde{A}_2\ldots\tilde{A}_{p-1}\tilde{A}_p\tilde{B}_p\tilde{A}_p + \ldots + \tilde{A}_2\tilde{B}_2\tilde{A}_2\tilde{A}_3\ldots\tilde{A}_p$$
$$+ \tilde{B}_1\tilde{A}_1\tilde{A}_2\ldots\tilde{A}_p = 2p\tilde{A}_2\tilde{A}_3\ldots\tilde{A}_p\tilde{H}_{p+1} \tag{32}$$

and

$$\tilde{H}_m\tilde{A}_m = \tilde{A}_m\tilde{H}_{m+1},$$

$$\tilde{H}_{m+1}\tilde{B}_m = \tilde{B}_m\tilde{H}_m. \tag{33}$$

The eqs. (32) and (33) have been obtained by substituting eqs. (31) in the parasuperalgebra relations of order p, i.e. (27a), (27b) and (27d).

Now, with the following ansatz for $\tilde{H}_m$, $m = 1, 2, \ldots, p+1$,

$$\tilde{H}_m = \frac{1}{2}\tilde{A}_m\tilde{B}_m + \frac{1}{2}C_m I_{2\times 2} = \frac{1}{2}\begin{pmatrix} \dfrac{E(n,m)}{E(n,m-1)} A(m-1)B(m-1) + C_m & 0 \\ 0 & A(m)B(m) + C_m \end{pmatrix}, \quad m = 1, 2, \ldots, p$$

$$\tilde{H}_{p+1} = \frac{1}{2}\tilde{B}_p\tilde{A}_p + \frac{1}{2}C_p I_{2\times 2} = \frac{1}{2}\begin{pmatrix} \dfrac{E(n,p)}{E(n,p-1)} B(p-1)A(p-1) + C_p & 0 \\ 0 & B(p)A(p) + C_p \end{pmatrix} \tag{34}$$

the eqs. (33) are automatically satisfied for $m = p$. By using ansatz (34) and shape invariance relations (29) in eqs. (33) for $m = 1, 2, \ldots, p-1$, we get a recursion relation among coefficients C_m as

$$C_m - C_{m+1} = E(n,m+1) - E(n,m). \tag{35}$$

Similar to [2], by substituting the ansatz into the first relation of eqs. (32) and using the shape invariance relation (29), we determine the constant C_1 in terms of spectra $E(n,m)$ as

$$C_1 = \frac{1}{p}\left[(1-p)E(n,1) + E(n,2) + E(n,3) + \ldots + E(n,p)\right]. \tag{36}$$

Thus, by using eqs. (35) and (36), we obtain

$$C_m = \frac{1}{p}\sum_{m'=1}^{p} E(n,m') - E(n,m), \qquad m = 1, 2, \ldots, p \tag{37}$$

Substituting the spectrum $E(n,m)$ given by formula (10) in eq. (37), we get

$$C_m = \frac{p-2m+1}{2}\left(\frac{A(x)W'(x)}{W(x)}\right)' + \frac{p^2-3m^2+3m-1}{2}A''(x),$$
$$m = 1, 2, \ldots, p. \tag{38}$$

Also, it is obvious that operators $\tilde{H}_m$ have the following eigenvalue equations with spinor eigenstates $\Psi_{n,m}(\theta)$ as

$$\tilde{H}_m\Psi_{n,m-1}(\theta) = \tilde{E}\Psi_{n,m-1}(\theta), \; m = 1, 2, \ldots, p+1, \tag{39}$$

where

$$\tilde{E} = \frac{p-2n-1}{4}\left(\frac{A(x)W'(x)}{W(x)}\right)' + \frac{p^2-3n^2-3n-1}{12}A''(x), \tag{40}$$

i.e., $\tilde{H}_m$ are isospectrum operators with spinors as eigenstates. Defining $\Omega(\theta)$ as a column matrix with $2(p+1)$ rows

$$(\Omega(\theta))_m := \Psi_{n,m}(\theta), \quad m = 0, 1, \ldots, p, \tag{41}$$

we have

$$H\Psi(\theta) = \tilde{E}\Omega(\theta). \tag{42}$$

In the $\Omega(\theta)$ representation for parasupersymmetry algebra, $Q_1^m\Omega(\theta)$ and $Q_2^m\Omega(\theta)$, $m = 1, \ldots, p$ will be the eigenstates of H will eigenvalue given in eq. (40), because Q_1 and Q_2 commute with H. Therefore, scalar spinors $\Psi_{n,m}(\theta)$ form the bases for the representation of unitary parasupersymmetry algebra of order p.

As we have explained in Ref. [2], by selecting various $A(x)$, eq. (6) and its solutions reduce to the associated special functions in mathematical physics. Hence, functions $\psi_{n,m}(\theta)$ which are expressed in terms of special functions (such as Jacobi, hypergeometric, Laguerre, ... associated functions), are obtained by multiplying the associated special functions on $A^{1/4}(x)W^{1/2}(x)$ to make scalar spinors $\Psi_{n,m}(\theta)$. These scalar spinors represent $N = 1$ chiral supersymmetry algebra and unitary parasupersymmetry algebra of order p. One may obtain the explicit form of these spinors for a given master function $A(x)$ and weight function $W(x)$.

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